J. Fluid Mech. (1966), *vol.* 26, *part* 2, *pp.* 255–266
Printed in Great Britain

The flow in a turbulent boundary layer after a change in surface roughness

By A. A. TOWNSEND
Emmanuel College, Cambridge

(Received 5 November 1965)

The changes of surface stress in a deep boundary layer passing from a surface of one roughness to another of different roughness are described fairly accurately by theories that assume self-preserving development of the flow modifications. It has been shown that the dynamical conditions for self-preserving flow can be satisfied if the change in friction velocity is small and if $\log l_0/z_0$ is large (l_0 is the depth of the modified flow and z_0 is the roughness length of the surface). In this paper it is shown that, if the change of friction velocity is not small, the dynamical conditions can be satisfied to a good approximation over considerable fetches if $\log l_0/z_0$ is large. The flow modification is then locally self-preserving, that is, the fields of mean velocity and turbulence are in a moving equilibrium but one which changes very slowly with fetch and depends on the ratio of the initial to the current friction velocity. In the limit of a very large increase in friction velocity, the moving equilibrium is essentially that of a boundary layer developing in a non-turbulent free stream. Equations describing the flow development are derived for all changes of friction velocity, and the form of the velocity changes is discussed. For large increases of friction velocity, the depth of the modified layer is substantially less than would be expected from the theories of Elliott and of Panofsky & Townsend.

1. Introduction

The changes in a turbulent boundary layer that passes from one surface to another of different roughness have been the subject of several experimental and theoretical investigations in recent years. The theories of Elliott (1958) and of Panofsky & Townsend (1964), which describe the observational material fairly well, assume self-preserving development of the flow modification induced by the change of surface, and it has been shown (Townsend 1965*a*) that the development is consistent with the Reynolds equations for mean flow momentum and turbulent energy if (i) the change in friction velocity is small, and (ii) $\log l_0/z_0$ is large (l_0 is the depth of the modified flow and z_0 is the roughness length). In meteorological situations, the change of roughness may be so large that the first condition is not satisfied and then the validity of the predictions may be queried. If the flow takes place from a very smooth to a very rough surface, the modified flow will resemble that in a turbulent boundary layer growing on the rough surface with a free stream of constant velocity, and it seems likely to be self-preserving. For less violent changes of surface or for a change from a rough to a smooth surface, the possibility of self-preserving development is not evident, but it will be shown that the development is *almost* self-preserving whatever the change of friction velocity.

2. Self-preserving forms for the flow modification

Consider a deep boundary layer, with zero longitudinal pressure gradient, flowing in the direction of the Ox axis. At $x = 0$, the surface roughness-length changes from its upstream value of z_1 to the downstream value, z_0. The notation and co-ordinates are those used in the earlier work, i.e.

Ox is in the wind direction,
z_1 is the roughness-length for $x < 0$,
z_0 is the roughness-length for $x > 0$,
$M = \log z_1/z_0$,
$U_1(z)$ is the mean velocity at height z for $x < 0$,
$U(x,z)$ is the mean velocity at (x,z),
$\delta(x,z)$ is an approximation to the net displacement of the streamline through (x,z),
u_1 is the friction velocity for $x < 0$,
τ_0 is the surface stress for $x > 0$,
u_0, v_0 are scales of velocity at fetch x,
l_0 is a scale of length at fetch x,
$\eta = z/l_0$,
k is the Karman constant, nearly 0·41.

Upstream of the change of roughness, the Reynolds stress is nearly independent of height with a kinematic value of u_1 and the velocity distribution is

$$U_1 = (u_1/k)\log(z/z_1), \tag{2.1}$$

where the suffix 1 indicates that the quantity refers to conditions upstream of the change of roughness. The suffix 0 refers to conditions downstream of the change of roughness but u_0 is used for the local velocity scale and *not* for the local friction velocity.

In the earlier paper (Townsend 1965*a*, to be called I), the velocity field $U(x,z)$ was defined using a distribution function $V(x,z)$ and the equations,

$$U = U_1 + V - u_1\delta(z)/(kz), \tag{2.2}$$

$$\delta(z) = -\frac{k}{u_1}(\log l_0/z_1 - C_0)^{-1}\int_0^z V(z')\,dz', \tag{2.3}$$

$$C_0 = -\int_0^\infty V(z)\log z/l_0\,dz\bigg/\int_0^\infty V(z)\,dz, \tag{2.4}$$

where l_0 is a measure of the depth of the region of modified flow, and U_1 is the upstream distribution of velocity. In I, V and δ were identified as the velocity change along a streamline of the mean flow distant z from the surface for negative x, and the net displacement of that streamline, but it was not made clear that they are approximations that are good only in the outer part of the flow. The point is of importance for the extension of the self-preserving theory to flows with large changes of roughness, and the following treatment may be clearer than the original in I.

The actual streamline displacement in the flow, δ_x, is given by

$$\int_0^z U_1(z')\,dz' = \int_0^{z+\delta_x} U(z')\,dz', \tag{2.5}$$

or, if δ_x/z is small (an assumption to be confirmed later), by

$$\delta_x U(z) = -\int_0^z (U-U_1)\,dz' \tag{2.6}$$

with a fractional error of order δ_x/z. Substituting for $(U-U_1)$ from equation (2.2),

$$\int_0^z \left[V(z') - \frac{u_1\delta(z')}{kz'}\right] dz' = -\delta_x \frac{u_1}{k}\left[\log\frac{z}{z_1} - \frac{\delta}{z} + \frac{kV}{u_1}\right]. \tag{2.7}$$

After an integration by parts and use of equation (2.3), we obtain

$$\delta_x\left[\log\frac{z}{z_1} - \frac{\delta}{z} + \frac{kV}{u_1}\right] = \delta\left[\log\frac{z}{z_1} - C_0 + C_0(z)\right], \tag{2.8}$$

where
$$C_0(z) = -\int_0^z V(z')\log\frac{z'}{l_0}\,dz' \bigg/ \int_0^z V(z')\,dz'$$

and so, for $|C_0 - C(z)| \ll \log z/z_1$,

$$(\delta_x - \delta)/\delta_x = \{\delta/z - kV/u_1 - C_0 + C_0(z)\}/(\log z/z_1). \tag{2.9}$$

The distribution function must approach zero for large values of z/l_0 and, for small values, it must conform with the logarithmic distribution of velocity in the equilibrium layer,

$$U = (\tau_0^{\frac{1}{2}}/k)\log z/z_0. \tag{2.10}$$

The last is possible if

$$V = (u_0/k)\,[\log z/l_0 + C] \tag{2.11}$$

for small z/l_0, where C is a constant of order one depending on the choice of l_0. It is then easy to show that the logarithmic forms are consistent with the defining equations (2.2–4) only if

$$\tau_0^{\frac{1}{2}} = u_1 + u_0[1 + \{\log(l_0/z_0) - M - C_0\}^{-1}] \tag{2.12}$$

and
$$u_0/u_1 = -M\{\log(l_0/z_0) - C + 1\}^{-1}, \tag{2.13}$$

where $M = \log z_0/z_1$. The terms in the numerator on the right of (2.9) may now be estimated. Inside the equilibrium layer, the form (2.11) is valid and may be used to show that

$$\left.\begin{aligned} C_0(z) &\approx -\log z/l_0, \\ \delta/z &\approx M\log z/l_0[\{\log(l_0/z_0) - M\}\log l_0/z_0]^{-1} \\ kV/u_1 &\approx -M(\log z/l_0/\log l_0/z_0. \end{aligned}\right\} \tag{2.14}$$

If z/l_0 is very small, none of these is necessarily small compared with log z/z_1 and δ is not necessarily a good approximation to δ_x, but, near the outer edge of the equilibrium layer and generally in the modified flow where z/l_0 is not too small,

$$\left.\begin{aligned} C_0(z) &= O(1), \\ \delta/z &= O[M\{(\log(l_0/z_0)-M)\log l_0/z_0\}^{-1}], \\ kV/u_1 &= O[M/\log(l_0/z_0)]. \end{aligned}\right\} \quad (2.15)$$

For positive M, the matching relation (2.12) shows that u_0 is negative and it may not exceed u_1 in magnitude if the surface stress remains positive. Then $\log l_0/z_0$ is greater than M and the three terms of the numerator are all small compared with $\log z/z_1 = \log(z/l_0)+\log(l_0/z_0)-M$ if $\log l_0/z_0$ is large and z/l_0 is not too small. For negative M, $\log z/z_1$ is certainly more than $-M$, and we reach the same conclusion. It follows that $(\delta_x-\delta)/\delta_x$ is of order $\log l_0/z_0$ or smaller in the outer part of the modified flow, whatever the value of the change-of-roughness parameter M. δ defined by (2.3) is therefore a good approximation to the actual displacement of the streamlines and is always moderately small compared with z. Then equation (2.2) shows that V differs from the true change of velocity along the streamline by an amount of order $u_0/k(\log l_0/z_1)^{-2}$, and mean flow accelerations in the outer flow can be found using V. In the equilibrium layer, where z/l_0 may be very small, δ is not a good approximation to the streamline displacement and V is not a good approximation to the velocity change, but here the nature of the flow is determined by the local surface stress and is substantially unaffected by the flow acceleration. In this region, V and δ are merely convenient functions for describing the flow, convenient because they approximate closely to the velocity change and the streamline displacement where the flow acceleration matters, outside the equilibrium layer. It may be mentioned that the expressions for the added momentum flux in § 3 of I do not depend at all on the correspondence between V and δ and the velocity change and streamline displacement.

We now introduce the self-preserving form for the change of velocity,

$$V = (u_0/k)f(z/l_0), \quad (2.16)$$

where u_0, l_0 are functions of x only. For self-preserving development, all the mean values that describe the flow modification must be expressible in similar self-preserving forms with the same length-scale l_0. The stress modification should have the form,

$$\tau-u_1^2 = (\tau_0-u_1^2)F(z/l_0), \quad (2.17)$$

where $F(z/l_0)$ approaches one for small z/l_0 and is zero for large values of z/l_0. Other quantities are the turbulent energy $\frac{1}{2}\overline{q^2}$, the divergence of the transverse flux of turbulent energy $\partial(\overline{pw}+\frac{1}{2}\overline{q^2w})/\partial z$, and the rate of dissipation of turbulent energy by viscosity ϵ.

$$\left.\begin{aligned} \overline{q^2}-\overline{q_1^2} &= \frac{\tau_0-u_1^2}{u_1^2}\overline{q_1^2}\,Q(z/l_0), \\ \frac{\partial}{\partial z}(\overline{pw}+\tfrac{1}{2}\overline{q^2w}) &= \frac{u_d^3}{l_0}D(z/l_0), \\ \epsilon-u_1^3/(kz) &= \{(\tau_0^{\frac{3}{2}}-u_1^3)/kz\}\,E(z/l_0), \end{aligned}\right\} \quad (2.18)$$

in which the scales have been chosen so that the values appropriate to an equilibrium layer can be assumed for small values of z/l_0.

3. Possibility of self-preserving development

The basis of self-preserving development is a moving equilibrium in which the various processes of turbulent transport and eddy interactions combine to produce flow structures which at all stages are similar in form. The exact forms depend on the dynamics of the flow which is partially described by the Reynolds equation for the mean flow momentum,

$$U\frac{\partial U}{\partial x}+W\frac{\partial U}{\partial z}=\frac{\partial \tau}{\partial z},$$

and by the Reynolds equation for the turbulent kinetic energy,

$$-\tau\frac{\partial U}{\partial z}+U\frac{\partial(\frac{1}{2}\overline{q^2})}{\partial x}+W\frac{\partial(\frac{1}{2}\overline{q^2})}{\partial z}+\frac{\partial}{\partial z}(\overline{pw}+\tfrac{1}{2}\overline{q^2w})+\epsilon=0.$$

Consider first the result of substituting the self-preserving forms for the velocity change and the stress change, (2.16) and (2.17), in the momentum equation. The purpose is to assess the possibility of self-preserving development and so there is no need to consider the flow for small values of z/l_0. This part of the flow is an equilibrium layer where production and dissipation of turbulent energy are so intense that mean velocity is described by the logarithmic 'law of the wall', and fulfilment of the matching conditions (2.11), (2.12), (2.13) is enough to make self-preserving flow a dynamical possibility. Outside the equilibrium layer, the adjustment time of the turbulent motion is appreciable and advection of momentum and energy have an effect in the outer region which occupies perhaps four-fifths of the whole region of modified flow. Supposing $\log l_0/z_0$ to be large, the velocity change V is small compared with the local velocity and terms quadratic in V may be omitted. The result of the substitution is then

$$u_1\log\frac{z}{z_1}\left[\frac{du_0}{dx}f-\frac{u_0}{l_0}\frac{dl_0}{dx}\eta f'\right]+\frac{u_1}{l_0}\left[u_0\frac{dl_0}{dx}f-\frac{d(u_0l_0)}{dx}\eta^{-1}\int_0 f\,d\eta\right]=k^2\frac{\tau_0-u_1^2}{l_0}F', \tag{3.1}$$

where $\eta=z/l_0$ and primes denote differentiation with respect to η. From equation (2.13), the ratio of du_0/dx to $u_0/l_0\,dl_0/dx$ is

$$\frac{l_0}{u_0}\frac{du_0}{\partial x}=-(\log l_0/z_0-C+1)^{-1}$$

which is small, and $\log z/z_1$ is much larger than one in the outer region. To the approximation of large $\log l_0/z_0$, the momentum equation is

$$-(\log l_0/z_0-M)\,u_1u_0(dl_0/dx)\,\eta f'=k^2(\tau_0-u_1^2)\,F' \tag{3.2}$$

and, after using the relations (2.12) and (2.13), it is

$$-\frac{(\log l_0/z_0-M)(\log l_0/z_0-C+1)}{2(\log l_0/z_0-C+1)-M}\frac{dl_0}{dx}\eta f'=k^2F'. \tag{3.3}$$

If self-preserving development is consistent with the momentum equation, the coefficient of $\eta f'$ in (3.3) must be independent of x. Integration over all η gives

$$\frac{dl_0}{dx}\frac{(\log l_0/z_0-M)(\log l_0/z_0-C+1)}{2(\log l_0/z_0-C+1)-M} = k^2/I_1, \tag{3.4}$$

where
$$I_1 = -\int_0^\infty f(\eta)\,d\eta,$$

since $F(0) = 1$ by definition. If l_0 satisfies this development equation, the momentum equation reduces to the non-dimensional, self-preserving form

$$\eta f' = -I_1 F' \tag{3.5}$$

and it might appear that self-preserving development is possible if $\log l_0/z_0$ is large without qualification on the change in friction velocity. For very large negative values of M, the stress ratio τ_0/u_1^2 is large and the flow over the rougher downstream surface must resemble closely a turbulent boundary layer initiated at $x = 0$ with a free-stream velocity equal to the current value of $U_1(l_0)$. The nature of flow with an almost non-turbulent ambient flow is very different from the perturbation type flow for small values of M, and the boundary-layer flow must have different distribution functions from the perturbation flow. In these circumstances, it is not surprising that the question of self-preserving development for moderate values of u_0/u_1 is not simple.

In a turbulent flow, the mean flow and the turbulent motion interact, the Reynolds stresses accelerating the mean flow and the turbulent motion deriving its energy from the working of the mean flow against the Reynolds stresses. The Reynolds momentum equation describes the first of the processes and it has now been shown that a self-preserving velocity change is consistent with a self-preserving change of Reynolds stresses. To show that the self-preserving stress change can arise from the interaction between the mean flow and the turbulence, it is necessary to look at the energy equation. With the self-preserving distributions of (2.18) and to the approximation of large $\log l_0/z_0$, the energy equation is

$$-(\tau_0-u_1^2)\frac{u_1}{kz}F-\frac{u_1^2u_0}{kl_0}f'-\frac{u_0(\tau_0-u_1^2)}{kl_0}Ff'$$
$$-\frac{(\tau_0-u_1^2)\overline{q_1^2}}{ku_1}\frac{1}{l_0}\frac{dl_0}{dx}\eta Q'+\frac{u_d^3}{l_0}D+\frac{\tau_0^{\frac{3}{2}}-u_1^3}{kz}E = 0. \tag{3.6}$$

For this to reduce to a self-preserving form, the various coefficients must either be negligible or maintain constant ratios with variation of x. For values of u_0/u_1 that are neither small nor very large, the ratios of $\tau_0^{\frac{3}{2}}-u_1^3$, $u_1(\tau_0-u_1^2)$ and $u_0(\tau_0-u_1^2)$ must remain constant, possible only if u_0/u_1 is invariant, which is inconsistent with variation of l_0 and equation (2.13). It appears then that self-preserving development is possible only if

(A) $|u_0/u_1|$ is small with $\tau_0-u_1^2 = 2u_1u_0$ and $u_d^3 = u_1^2u_0$,

or (B) $u_0 \gg u_1$ with $\tau_0 = u_0^2$ and $u_d = u_0$.

The perturbation flows fall in group A and the boundary-layer type flow in group B (see figure 1).

While only the two extreme kinds of flow can remain strictly self-preserving over large ranges of x, flows with moderate values of u_0/u_1 can satisfy the criterion for self-preserving development over fetches long enough for the flow to attain a moving equilibrium. The reason is that the downstream variation of u_0 is very slow if $\log l_0/z_0$ is large. For example, with $\log l_0/z_0$ about ten, u_0 decreases by about one-fifth while l_0 (and x) increases by a factor of ten. The 'settling-down'

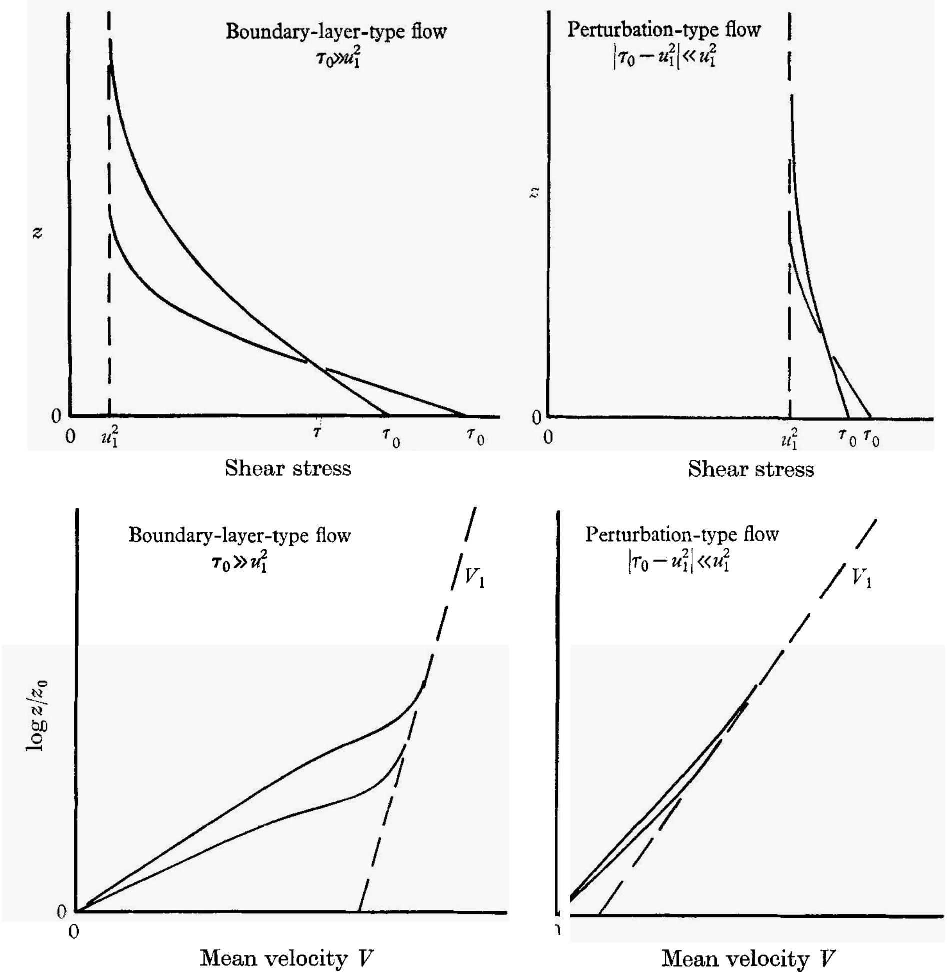

FIGURE 1. Velocity and stress distributions for a boundary-layer type flow and for a perturbation-type flow. (The distributions are schematic.)

time of a turbulent flow is comparable with the turbulent energy divided by the rate of energy dissipation which is about l_0/u_1. In this time, the parcel of turbulence is swept a distance of about $l_0/k \log l_0/z_1$, which, by equation (3.4) is comparable with x. It is therefore likely that the turbulent flow is always near the moving equilibrium appropriate to the current value of u_0/u_1 and that the flow is locally self-preserving. Over extensive ranges of x, the nature of the moving equilibrium changes and with it the form of the distribution functions, but they change so slowly that, at least to the first order, the variation can be neglected.

4. Development of the modified flow

In the previous section it was argued that the development is very nearly self-preserving although the forms of the distribution functions depend on the value of u_0/u_1 and are not strictly invariant during development. Then the modifications of mean velocity and Reynolds stress are described by the downstream variation of the scale length l_0 and by the form of the velocity distribution function $f(\eta)$. To a first-order approximation, the downstream variation of l_0 may be found by integrating the development equation (3.4), using the value of I_1 appropriate to the current value of u_0/u_1. However, we have not yet defined l_0 precisely and are free to choose it so that $I_1 = 1$ for all values of u_0/u_1, i.e. to put

$$l_0 = -\frac{k}{u_0}\int_0^\infty V(z)\,dz \tag{4.1}$$

with an obvious gain of convenience. Neglecting terms of order $(\log l_0/z_0)^{-1}$, l_0 is related to x by

$$l_0\frac{\log l_0/z_0(\log l_0/z_0-M)}{2\log l_0/z_0-M} = k^2(x-x_0), \tag{4.2}$$

where x_0 is an effective origin of the modified flow.

For the purpose of predicting flow changes when $\log l_0/z_0$ is only moderately large, it is desirable to have a better approximation for l_0 than equation (4.2). If P_x is the additional momentum flux in the modified flow, overall conservation of momentum requires that

$$dP_x/dx = u_1^2-\tau_0.$$

Omitting terms of order $Mu_1^2l_0(\log l_0/z_0)^{-2}$, it can be shown (see Townsend 1965*a*) that

$$P_x = \frac{u_1}{k}(\log l_0/z_1-C_0)\int_0^\infty V\,dz+\int_0^\infty (V(z))^2\,dz$$

and so, substituting $V = (u_0/k)f(z/l_0)$,

$$P_x = \frac{Mu_1^2l_0}{k^2}\left[\frac{\log l_0/z_0-M-C_0}{\log(l_0/z_0)-C+1}+\frac{MI_2}{(\log l_0/z_0-C+1)^2}\right], \tag{4.3}$$

where

$$I_2 = \int_0^\infty (f(\eta))^2\,d\eta.$$

The quantities C, C_0 and I_2 depend on the current value of u_0/u_1 and, to some extent, on the current value of $\log l_0/z_0$, but they do not affect the leading terms of P_x and the effect of their variations on the magnitude of dP_x/dx is of order $Mu_1^2(dl_0/dx)(\log l_0/z_0)^{-2}$. Then

$$\frac{dP_x}{dx} = \frac{Mu_1^2}{k^2}\frac{dl_0}{dx}\left[\frac{\log l_0/z_0-M-C_0}{\log l_0/z_0-C+1}+\frac{M(1+I_2)}{(\log l_0/z_0-C+1)^2}\right]. \tag{4.4}$$

From (2.6),

$$u_1^2-\tau_0 = Mu_1^2\left[2\frac{\log(l_0/z_0)-M-C_0+1}{\{\log(l_0/z_0)-C+1\}\{\log l_0/z_0)-M-C_0\}} - M\frac{\log l_0/z_0-M-C_0+2}{\{\log(l_0/z_0)-C+1\}^2\{\log(l_0/z_0)-M-C_0\}}\right]$$

and we obtain the development equation,

$$k^2 = \frac{dl_0}{dx}\left[\frac{\{\log(l_0/z_0)-C+1\}\{\log(l_0/z_0)-M-C_0\}+M(1+I_2)}{2\{\log(l_0/z_0)-C+1\}-M} - \tfrac{1}{2} + \frac{\tfrac{1}{2}M^2}{(2\log(l_0/z_0)-M)^2}\right] \quad (4.5)$$

which may be integrated to

$$\frac{k^2(x-x_0)}{l_0} = \frac{\{\log(l_0/z_0)-C+1\}\{\log(l_0/z_0)-M-C_0\}+M(1+I_2)}{2\{\log(l_0/z_0)-C+1\}-M} - 1. \quad (4.6)$$

The omitted terms are less in a ratio of order $(\log l_0/z_0)^{-2}$ than those retained.

In the two limiting flows, the development equation takes simple forms. For small ratios of the friction velocities, it becomes (compare Townsend 1965*b*)

$$k^2(x-x_0)/l_0 = \tfrac{1}{2}\{\log(l_0/z_0) - \tfrac{1}{2}M - C_0 - 2\} \quad (4.7)$$

valid for $|M| \ll \log l_0/z_0$, while for large positive ratios, i.e. $-M \gg \log l_0/z_0$, it is

$$k^2(x-x_0)/l_0 = \log(l_0/z_0) - C - 1 - I_2 + (\log l_0/z_0)^2/M. \quad (4.8)$$

As defined, the depth of the modified layer is seen to be nearly twice as large at a given fetch in the perturbation flow as in the boundary-layer type flow. This

Velocity profile	C	C_0	I_2	δ_m/l_0
Boundary layer	−0·60	1·5	2·9	0·55
Elliott	0	2	2	0·90
Panofsky–Townsend	0·31	1·81	1·67	1·22

TABLE 1. Characteristics of velocity distribution functions

conclusion takes no account of possible changes in shape of the velocity distribution function $f(\eta)$, whose form is known with confidence only for the boundary layer. Some measurements are collected in Townsend (1956) and, in terms of the scale thickness used there, $\delta = z_0 \exp(kU_1/u_0)$ in the present notation,

$$I_1 = 0{\cdot}55\delta/l_0 = 1.$$

Figure 2 shows the form of the velocity distribution function and it can be seen that the limit of observable velocity change lies near $z = 0{\cdot}55 l_0$. The values of the parameters, C, C_0 and I_2, are given in table 1.

Observations of the velocity changes have been made by several workers (Lettau *et al.* 1962; Rider, Philip & Bradley 1963; Bradley 1965) in the atmospheric boundary layer, mostly for moderate values of u_0/u_1 near 0·7. The accuracy of the observations is hardly sufficient to do more than indicate the general form of $f(\eta)$ and the following reasoning may be as good a guide to the form as the observations. An essential difference between the perturbation flows with small u_0/u_1 and the boundary-layer flows is that, in the first kind, the ambient flow is already turbulent and able to convect turbulent energy in the

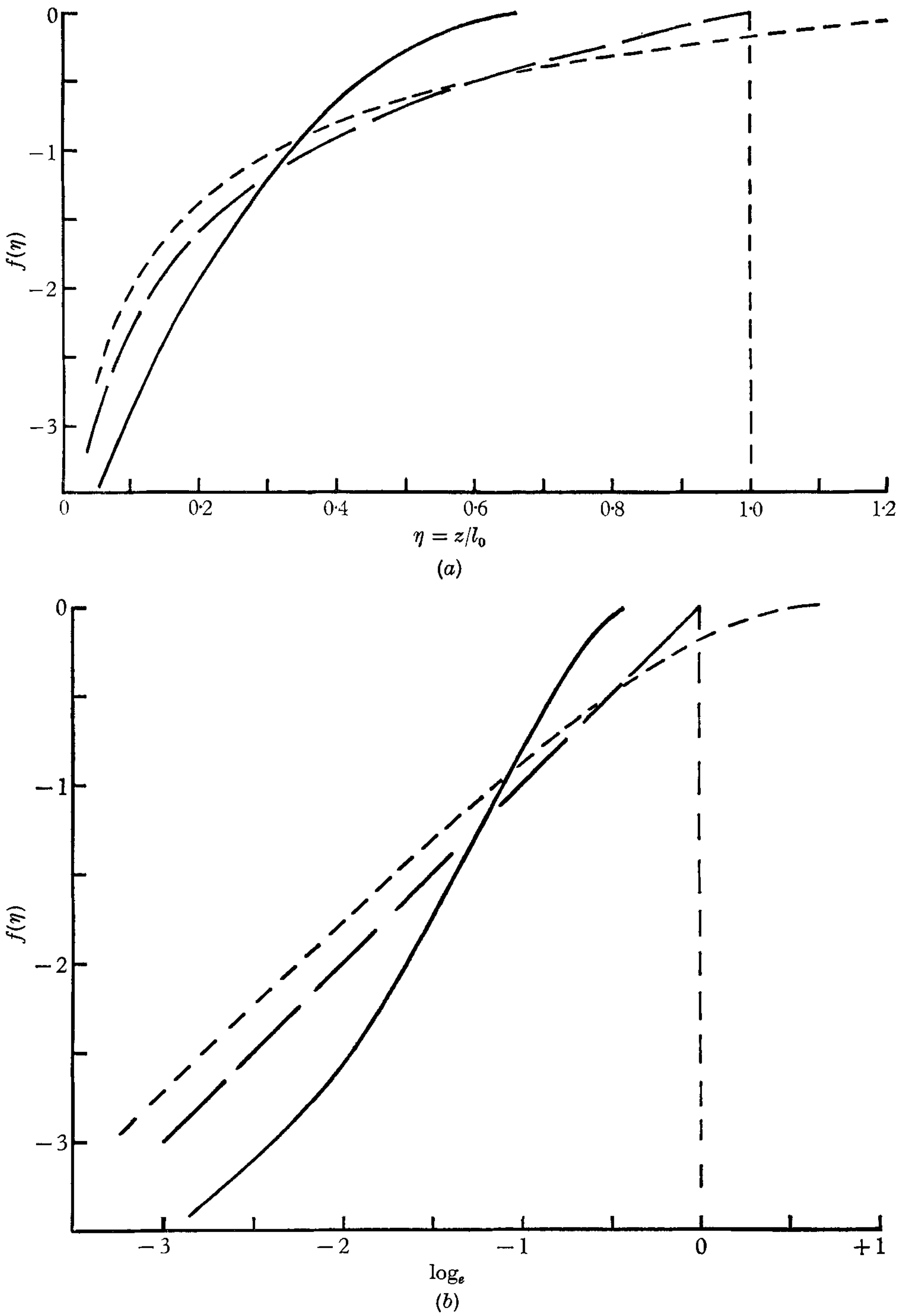

FIGURE 2. Comparison of profile shapes as observed in boundary layers and as postulated by Elliott and by Panofsky & Townsend, (*a*) with a linear height scale, and (*b*) with a logarithmic height scale. ——, Boundary-layer profile; — —, Elliott (logarithmic) profile; - - - -, Panofsky–Townsend (log-linear) profile.

lateral direction. The other processes represented in the energy equation, production, dissipation and advection of turbulent energy, are qualitatively similar in both kinds of flow and it may be expected that the change in form of the velocity distribution function is, for the most part, a response to the change of magnitude of lateral convection of turbulent energy. Common sense, reinforced perhaps by theoretical models of the kind used to discuss the perturbation flow (Townsend 1965*a*), is enough to show that added lateral convection leads to a relatively increased spread of the flow modification, and so that the depth of the modified region, expressed as a fraction of l_0, becomes less as u_0/u_1 increases. For small values of u_0/u_1, it is likely that the distribution function is close to the logarithmic form, proposed by Elliott (1958),

$$\begin{aligned} f(\eta) &= \log\eta \quad \text{for} \quad \eta < 1 \\ &= 0 \quad \text{for} \quad \eta > 1, \end{aligned}$$

although the Panofsky & Townsend (1964) form,

$$\begin{aligned} f(\eta) &= \log\tfrac{1}{2}\eta + (1-\tfrac{1}{2}\eta) \quad \text{for} \quad \eta < 2 \\ &= 0 \quad \text{for} \quad \eta > 2. \end{aligned}$$

is possible. Table 1 gives values of C, C_0 and I_2 for these profiles, and also the total effective thickness of the modified layer δ_m, defined as the height at which the velocity change is $\frac{1}{4}u_0$.

In view of the uncertainty in the basic velocity distribution, it may be useful to have a summary of the consequent uncertainty in prediction of stress and depth of the modified layer. The following estimates refer to fetches such that $\log l_0/(z_1 z_0)^{\frac{1}{2}} = \log l_0/z_0 - \frac{1}{2}M$ is near six.

(i) For a given $\log l_0/(z_1 z_0)^{\frac{1}{2}}$, the predicted scale height l_0 varies over a range of 6 % depending on the assumed profile.

(ii) The effective depth of the modified layer, i.e. δ_m as defined above, is about one-half of l_0 for large negative M, but may be nearly equal to l_0 for small or positive values of M.

(iii) The fractional increase in friction velocity, u_0/u_1, depends on the assumed profile and has a consequent uncertainty of about 10 %.

5. Concluding remarks

The conclusion to be drawn from the analysis is that the flow modification induced by a change of roughness can be self-preserving in form over fetches long compared with the adjustment length of the flow. For changes of friction velocity that are not small, the dynamics of the self-preserving flow change slowly with fetch and cause slow changes in the distribution functions for velocity and stress, but the flow will always be very near the hypothetical self-preserving state appropriate to the current value of the stress-ratio if the change of flow velocity is small over most of the modified layer. The condition for this is that both $\log l_0/z_0$ and $\log l_0/z_1$ are moderately large. Since measurements are usually made at heights greater than the physical height of the roughness elements (which are perhaps twenty times as large as the roughness length), the condition is satisfied for any value of M, the change of roughness parameter.

The predictions are to some extent dependent on the forms of the distribution functions, but the available observations are not of sufficient accuracy to determine the variation of form with stress-ratio.

REFERENCES

BRADLEY, E. F. 1965 Ph.D. dissertation, Australian National University.

ELLIOTT, W. P. 1958 *Trans. Amer. Geophys. Union* **39**, 1048.

LETTAU, H. H. *et al.* 1962 *U.S. Army Electronic Proving Ground Annual Rep.*, DA 36-039-SC-80282.

PANOFSKY, H. A. & TOWNSEND, A. A. 1964 *Quart. J. Roy. Met. Soc.* **90**, 147.

RIDER, N. E., PHILIP, J. R. & BRADLEY, E. F. 1963 *Quart. J. Roy. Nat. Soc.* **89**, 507.

TOWNSEND, A. A. 1956 *The Structure of Turbulent Shear Flow.* Cambridge University Press.

TOWNSEND, A. A. 1965*a* *J. Fluid Mech.* **22**, 773.

TOWNSEND, A. A. 1965*b* *J. Fluid Mech.* **22**, 799.